CH: 2 GRAVITATION

1) Gravitational force of attraction between two point masses.

$$F = G.\frac{m_1 m_2}{r^2}$$

2) Acceleration due to gravity:

(a) On earth surface:

$$g = \frac{GM}{R^2}$$

(b) At height 'h' above earth:

$$g_h = \frac{GM}{(R+h)^2} = g\left(\frac{R}{R+h}\right)^2$$

If $h << R$: $g_h = g\left(1 - \frac{2h}{R}\right)$

(c) At depth 'd' below earth:

$$g_d = g\left(1 - \frac{d}{R}\right)$$

(d) At a latitude 'ϕ':

$$g' = g - R\omega^2 \cos^2\phi$$

① At equator: $\phi = 0$

② At poles: $\phi = 90°$

3) Critical velocity of satellite:

$$V_c = \sqrt{\frac{GM}{r}} = \sqrt{g_h (R+h)}$$

4) Time period of satellite:

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{r}{g_h}}$$

⑩ Lift: [Reason in Lift]

a) Moving up → $W = mg + ma$

b) Moving down ⇒ $W = mg - ma$

5) Binding energy:

(a) Stationary body on earth surface:

$$B.E. = \frac{GMm}{R}$$

$P.E. = -\frac{GMm}{r}$

$K.E. = \frac{GMm}{2r}$

(b) Revolving around earth: $B.E. = \frac{1}{2}.\frac{GMm}{r}$

6) Escape velocity: $V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$

7) Intensity of gravitational field: $I = \frac{F}{m}$

8) Kepler's third Law (Law of period):

$$T^2 \propto r^3$$

9) Gravitational Potential: $V_P = -\frac{GM}{r}$

a) At earth surface: $r = R$ $\therefore V_P = -\frac{GM}{R}$

b) at $r=\infty$, $V_P = 0$ (At infinity)
~~c) at centre of earth: r = 0, $V_P = 0$ (Minimum)~~

① Two spheres of masses m_1 & m_2 and separated by a distance 'd' are situated in air & the gravitational force of attraction between them is F. If the two spheres are kept in a liquid of specific gravity s then the gravitational force between them will be ___ [AIPMT'2003]

a) SF b) $\frac{F}{S}$ c) F d) $\sqrt{F}$

⇒ (c) $F = G.\frac{m_1 m_2}{r^2}$

∴ F is independent of medium, in which the two bodies are kept.
∴ Gravitational force = F

② The values of the acceleration due to gravity on two planets are g_1 & g_2. If $g_1 = g_2$, then the two planets must have the same ___

a) radius b) mass
c) (mass/radius)^2 d) mass/(radius)^2

⇒ (d) $g = \frac{GM}{R^2}$

If $g_1 = g_2$ then $\frac{GM_1}{R_1^2} = \frac{GM_2}{R_2^2}$

$\therefore \frac{M_1}{R_1^2} = \frac{M_2}{R_2^2} = \frac{\text{mass}}{(\text{radius})^2}$

③ If G is the universal constant of gravitation & g is the acceleration due to gravity, then the dimensions of $\frac{G}{g}$ are ___

a) $L^1M^1T^0$ b) $L^2M^{-1}T^0$ c) $L^{-2}M^1T^2$ d) $L^3M^{-2}T^1$

⇒ (b) $g = \frac{GM}{R^2}$ ∴ $\frac{G}{g} = \frac{R^2}{M}$

$= \frac{m^2}{kg} = [L^2 M^{-1} T^0]$

4) If the earth suddenly shrinks to half of its present size, then the acceleration due to gravity will be —

a) $\frac{g}{2}$ b) $\frac{g}{4}$ c) $2g$ d) $4g$

⇒ (d) $g = \frac{GM}{R^2}$ { ∵ R becomes half.

$\therefore g' = \frac{GM}{(R/2)^2} = \frac{4GM}{R^2} = 4g$

5) A force of 75 Newton acts on a body of mass 2.5 kg at a certain point. The intensity of the gravitational field at that point is —

a) 15 N/kg b) 30 N/kg c) 45 N/kg d) 20 N/kg

⇒ (b) Intensity of gravitational field

$I = \frac{F}{m} = \frac{75}{2.5} = 30 \text{ N/kg}$

6) What would be the acceleration due to gravity on the surface of a planet if its radius is 1/4 the radius of the earth & its mass is $1/80^{th}$ the mass of the earth?

a) $g_p = \frac{g}{2}$ b) $g_p = \frac{g}{5}$ c) $g_p = \frac{g}{3}$ d) $g_p = \frac{g}{4}$

⇒ (b) For earth ⇒ $g = \frac{GM}{R^2}$

For planet ⇒ $g_p = \frac{G(M/80)}{(R/4)^2}$

$\therefore g_p = \frac{GM}{80} \times \frac{16}{R^2} = \frac{1}{5} \times \frac{GM}{R^2} = \frac{g}{5}$

7) The time period of a simple pendulum inside a stationary lift is 2 second. What would be its period, when the lift moves upwards with an acceleration $g/4$?

a) 2 sec b) $\frac{4}{\sqrt{5}}$ sec c) $\sqrt{5}$ sec d) 4 sec

→ (b) $T_1 = 2\pi\sqrt{\frac{L}{g}}$ ⇒ Stationary lift

$T_2 = 2\pi\sqrt{\frac{L}{g+\frac{g}{4}}}$ ⇒ Moving upwards

$= 2\pi\sqrt{\frac{L}{5g/4}}$

$\therefore \frac{T_2}{T_1} = \frac{2\pi\sqrt{4L/5g}}{2\pi\sqrt{L/g}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

$\therefore T_2 = T_1 \times \frac{2}{\sqrt{5}} = 2 \times \frac{2}{\sqrt{5}} = \frac{4}{\sqrt{5}}$ sec

(8) A simple pendulum of length L is taken from earth to a planet where the acceleration due to gravity is doubled. What should be its new length so that its periodic time is not changed?

a) L b) 2L c) $\frac{L}{2}$ d) 3L

→ (b) $T = 2\pi\sqrt{\frac{L}{g}}$ ⇒ Earth

$T' = 2\pi\sqrt{\frac{L'}{2g}}$ ⇒ Planet $\left\{ \begin{array}{l} \text{New gravity} \\ = 2g \end{array} \right.$

$\because T = T'$

$\therefore 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{L'}{2g}}$

Squaring

$\therefore \frac{L}{g} = \frac{L'}{2g}$ $\therefore L' = 2L$

(9) The unit of g/G is —

a) kg/m b) kg/m^2 c) m^2/kg d) m/kg

→ (b) $g = \frac{GM}{R^2}$

$\therefore \frac{g}{G} = \frac{M}{R^2} = \frac{kg}{m^2}$

(10) The acceleration due to gravity on the moon is $1/6^{th}$ that on the earth. If the earth & the moon are assumed to have the same density, then the radius of the moon is –

a) $6 R_e$ b) $\frac{1}{6} R_e$ c) $\frac{1}{3} R_e$ d) $\frac{1}{4} R_e$

⇒ (b) $g = \frac{GM}{R^2} = \frac{G}{R^2}$ [Vol × Density]

$$= \frac{G}{R^2}\left(\frac{4}{3}\pi R^3 \rho\right)$$

$$\therefore g = \frac{4}{3}\pi R G \rho \quad (1) \Rightarrow \text{For earth}$$

$$\& \ \frac{g}{6} = \frac{4}{3}\pi R' G \rho \quad (2) \Rightarrow \text{For moon}$$

Eqⁿ (2) ÷ (1) ⇒ $\frac{g/6}{g} = \frac{4/3 \pi R' G \rho}{4/3 \pi R G \rho}$

$$\therefore \frac{1}{6} = \frac{R'}{R} \qquad \therefore R' = \frac{R}{6} = \frac{1}{6} R_e$$

(11) Two planets have the same average density but their radii are R_1 & R_2. If the acceleration due to gravity on these planets are g_1 & g_2 resp, then –

a) $\frac{g_1}{g_2} = \frac{R_1^3}{R_2^3}$ b) $\frac{g_1}{g_2} = \frac{R_2}{R_1}$ c) $\frac{g_1}{g_2} = \frac{R_1^2}{R_2^2}$ d) $\frac{g_1}{g_2} = \frac{R_1}{R_2}$

⇒ (d) $g = \frac{GM}{R^2} = \frac{G}{R^2}\left[\frac{4}{3}\pi R^3 \rho\right]$

$$= \frac{4}{3}\pi R G \rho$$

$$\therefore g \propto R$$

$$\therefore \frac{g_1}{g_2} = \frac{R_1}{R_2}$$

(12) A man can jump 2 m high on the surface of the earth. How high he can jump on a planet where the acceleration due to gravity is $\frac{g}{12}$, where g is the acceleration due to gravity on the surface of the earth?

a) 12 m   b) 18 m   c) 24 m   d) 30 m

⇒ (c) Potential energy is required for a man to jump.

∴ Energy $= mgh$ ⇒ Earth

$= mg'h'$ ⇒ Planet.

$$\therefore gh = g'h'$$

$$\therefore h' = \frac{gh}{g'} = \frac{g \times 2}{g/12} = 24\,m$$

(13) A man weighs 60 kg wt at the earth's surface. The radius of the earth is 6400 km. At what height above the earth's surface, his weight becomes 30 kg wt? [$\sqrt{2} = 1.414$]

a) 1000 km   b) 1500 km   c) 2650 km

d) 2900 km

⇒ (c) $g = \frac{GM}{R^2}$ ⇒ At earth surface

& $g' = \frac{GM}{(R+h)^2}$ ⇒ At height 'h'

Now, ∵ weight becomes half i.e. 60 kg wt to 30 kg wt & $W = mg$

$$\therefore g' = \frac{g}{2}$$

$$\therefore \frac{g'}{g} = \frac{R^2}{(R+h)^2} \qquad \therefore \frac{g/2}{g} = \frac{R^2}{(R+h)^2}$$

$$\therefore \frac{1}{2} = \frac{R^2}{(R+h)^2}$$

$$\therefore \left[\frac{R+h}{R}\right]^2 = 2$$

$$\therefore \left[1 + \frac{h}{R}\right]^2 = 2$$

$$\therefore 1 + \frac{h}{R} = \sqrt{2}$$

$$\therefore \frac{h}{R} = \sqrt{2} - 1$$

$$\therefore h = R(\sqrt{2} - 1)$$

$= 6400\,(1.414 - 1)$

$= 6400 \times 0.414 = 2650\ km$

(14) The weight of a man in a lift moving upwards with an acceleration 'a' is 600 N. When the lift moves downwards with the same acceleration his weight is found to be 360 N. The real weight of the man is —

a) 380 N b) 600 N c) 480 N d) 700 N

⇒ (c) When lift moves up | When lift moves down

$W_1 = mg + ma$ | $W_2 = mg - ma$

$\therefore 600 = mg + ma \quad —(1)$ | $360 = mg - ma \quad —(2)$

Eqⁿ (1) + (2) ⇒ $960 = 2mg$

$\therefore mg = \frac{960}{2} = 480$

$\therefore$ Real weight = 480 N

(15) The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g is acceleration due to gravity on the surface of the earth) in terms of R, where R is the radius of the earth is — [AIEEE' 2009]

a) $\frac{R}{\sqrt{2}}$ b) R c) $\sqrt{2}R$ d) 2R

→ (d) same as Q. No. (13)

$g = \frac{GM}{R^2}$ & $g' = \frac{GM}{(R+h)^2}$

$\therefore \frac{g'}{g} = \frac{R^2}{(R+h)^2}$

Now, $g' = g/9$

$\therefore \frac{g/9}{g} = \frac{R^2}{(R+h)^2}$

$\therefore \frac{1}{9} = \frac{R^2}{(R+h)^2}$

$\therefore (R+h)^2 = 9R^2$

Taking square root

$\therefore R+h = 3R$

$\therefore h = 3R - R = 2R$

(16) The density of a newly discovered planet is twice that of the earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth. If the radius of the earth be R, then radius of the planet would be –

a) $4R$ b) $\frac{R}{2}$ c) $\frac{R}{4}$ d) $2R$

$\Rightarrow$ (b) $g = \frac{GM}{R^2} = \frac{G}{R^2}\left[\frac{4}{3}\pi R^3 \times \rho\right]$

$\therefore g = \frac{4}{3}\pi G R \rho$

$\therefore g \propto R\cdot\rho$ ← Earth

$g = R_1 \times 2\rho$ ← Planet

$\therefore \frac{g'}{g} = \frac{R'\rho}{R\rho}$

$\therefore 1 = \frac{R' \times 2\rho}{R\rho}$

$\therefore R' = R/2$

$\left\{ g' = g \right.$ Given

$\therefore \frac{g}{g} = \frac{R\rho}{R_1 \times 2\rho}$

(17) A satellite is revolving around the earth in a circular orbit at an altitude 'h' where the acceleration due to gravity is g'. If the earth is a sphere of radius R then the period of the satellite is given by.

a) $T = 2\pi\sqrt{\frac{h}{g'}}$ b) $T = 2\pi\sqrt{\frac{R}{g'}}$

c) $T = 2\pi\sqrt{\frac{R+h}{g'}}$ d) $T = 2\pi\sqrt{\frac{R-h}{g'}}$

$\Rightarrow$ (C) Period of satellite is given by

$T = 2\pi\sqrt{\frac{r}{g_h}}$ $\left\{ g_h \to \right.$ Acceln at height 'h'

Now, Acceln at height h = g' ... Given

$\therefore T = 2\pi\sqrt{\frac{R+h}{g'}}$ $\{\because r = R+h$

(18) If the radius of the earth's orbit is made one-fourth of its present value, then the duration of the year will be ___

a) (365 × 8) days b) (365 × 4) days
c) $\left(\frac{365}{4}\right)$ days d) $\left(\frac{365}{8}\right)$ days

→ (d) $T^2 \propto r^3$ ... Kepler's law

$\therefore \frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3} = \left(\frac{r_2}{r_1}\right)^3$ $\{r_2 = \frac{r_1}{4}$

$\therefore T_2^2 = T_1^2 \times \left(\frac{r_1/4}{r_1}\right)^3$

$= (365)^2 \times \left(\frac{1}{4}\right)^3 = \frac{365^2}{64} = \frac{365^2}{8^2}$

$\therefore T_2 = \left(\frac{365}{8}\right)$ days

(19) Two satellites orbiting around the earth have their critical speeds in the ratio 4:5. What is the ratio of their orbital radii?

a) $\frac{5}{4}$ b) $\frac{4}{5}$ c) $\frac{25}{16}$ d) $\frac{12}{5}$

→ (c) $V_c = \sqrt{\frac{GM}{r}}$ $\therefore V_c \propto \sqrt{\frac{1}{r}}$

$\therefore \frac{V_{c1}}{V_{c2}} = \sqrt{\frac{r_2}{r_1}}$

$\therefore \frac{4}{5} = \sqrt{\frac{r_2}{r_1}}$ squaring

$\therefore \frac{r_2}{r_1} = \frac{16}{25}$ $\therefore \frac{r_1}{r_2} = \frac{25}{16}$

(20) The critical speed of a satellite of mass 500 kg is 20 m/s. What is the critical speed of a satellite of mass 1000 kg moving in the same orbit?

a) 0 m/s b) 20 km/hr c) 72 m/s d) 72 km/hr

⇒ (d) $V_c = \sqrt{\frac{GM}{r}}$ Here no term of 'm' present.

$\therefore V_c$ is independent of mass of satellite

$\therefore V_c = 20$ m/s

$= \frac{20 \times 3600}{1000}$ km/hr

$= 72$ km/hr

(21) A satellite is orbiting around the earth in a circular orbit. The centripetal force acting on it is 'F'. The gravitational force of the earth acting on the satellite is also equal to F. The net force acting on the satellite is —

a) 2F b) zero c) F d) $\frac{F}{2}$

⇒ (c) The gravitational force produces the centripetal force.

$\therefore$ Net force = F

(22) A satellite going around the earth suddenly loses height & start moving in a lower orbit. The speed of the satellite —

a) does not change b) is decreased

c) is increased d) may increase or decrease

⇒ (c) $V_c = \sqrt{\frac{GM}{r}}$ $\therefore V_c \propto \frac{1}{\sqrt{r}}$

$\therefore$ As r decreases, speed increases.

(23) A planet year is 8 times the earth year. How far is the planet from the sun, if the earth is 1.5×10^8 km away from the sun?

a) 3×10^8 km b) 4.5×10^8 km

c) 6×10^8 km d) 9×10^8 km

⇒ (c) As per Kepler's third law -

$$T^2 \propto r^3$$

$$\therefore \frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3} = \left(\frac{r_2}{r_1}\right)^3$$

$$\therefore \frac{r_2}{r_1} = \left(\frac{T_2}{T_1}\right)^{2/3}$$

$$= \left(\frac{8T_1}{T_1}\right)^{2/3} \quad \left\{\because T_2 = 8T_1\right.$$

$$= (2^3)^{2/3} = 2^2 = 4$$

$$\therefore r_2 = r_1 \times 4$$

$$= 1.5 \times 10^8 \times 4$$

$$= 6 \times 10^8 \text{ km}$$

24) A satellite of mass 'm' revolves around the earth of radius 'R' at a height 'x' from its surface. If 'g' is the acceleration due to gravity on the surface of the earth, then the orbital speed of the satellite is - [AIEEE'04]

a) $\left(\frac{gR^2}{R+x}\right)^{1/2}$ b) $\frac{gR}{R-x}$ c) gh d) $\frac{gR^2}{R+x}$

⇒ (a) $V_e = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+x}}$ $\left\{\begin{array}{l}\because \text{Radius} = R \\ \& \text{ height} = x\end{array}\right.$

Now, $g = \frac{GM}{R^2}$ $\therefore GM = gR^2$

$$\therefore V_e = \sqrt{\frac{gR^2}{R+x}} = \left(\frac{gR^2}{R+x}\right)^{1/2}$$